SECTION 9

ANGLES

Contents

9.1 Introduction 9–3

9.2 Coordinate Systems, Angles, and Unit Vectors at a Tracking Station on Earth 9–4

9.2.1 Hour Angle and Declination..9–5

9.2.2 The North-East-Zenith Coordinate System.................9–7

9.2.3 Azimuth and Elevation...9–10

9.2.4 X and Y Angles..9–11

9.2.5 X' and Y' Angles 9–13

9.2.6 Conversion of Earth-Fixed Unit Vectors to Space-Fixed Unit Vectors 9–14

9.3 Computation of Angles at Reception and Transmission Times at a Tracking Station on Earth 9–15

9.3.1 Unit Vector **L**..9–15

9.3.2 Refraction Correction $\Delta_r\gamma$ 9–20

9.3.2.1 Modified Berman-Rockwell Model................9–20

9.3.2.2 Lanyi Model 9–21

9.3.2.2.1 Equations...9–21

9.3.2.2.2 Atmospheric Parameters 9–23

9.3.3 Computed Angles 9–25

9.3.3.1 Hour Angle and Declination 9–25

9.3.3.2 Azimuth and Elevation 9–26

9.3.3.3 X and Y Angles 9–27

9.3.3.4 X' and Y' Angles 9–27

9.4 Corrections Due to Small Rotations of Reference Coordinate System at Tracking Station on Earth 9–28

9.5 Computation of Auxiliary Angles at Earth Satellites 9–30

9.5.1 Auxiliary Angles at Reception Time at TOPEX Satellite 9–30

9.5.2 Auxiliary Angles at Transmission Time at a GPS Satellite 9–31

Figures

9–1 Hour Angle and Declination 9–6

9–2 The North-East-Zenith Coordinate System 9–8

9–3 Azimuth and Elevation 9–11

9–4 X and Y Angles 9–12

9–5 X' and Y' Angles 9–14

9.1 INTRODUCTION

This section gives the formulation used to calculate auxiliary angles and the computed values of angular observables. Auxiliary angles are used in calculating the computed values of observables. They are used to calculate the antenna corrections described in Section 10. The auxiliary elevation angle is used to calculate the delay of the radio signal due to the troposphere. Auxiliary angles are used for data editing and data weighting and can also be used for antenna pointing predictions.

Section 9.2 describes antenna angles of the spacecraft (a free spacecraft or a landed spacecraft on a celestial body) or a quasar measured at a tracking station on Earth. Computed values of angular observables are measured at the reception time t_3 at the receiving station on Earth. For all data types, auxiliary angles are calculated at the reception time t_3 (denoted as t_1 and t_2 at receiving stations 1 and 2 on Earth for quasar interferometry data types) and also at the transmission time t_1 (for round-trip data types) at a tracking station on Earth. For each angle pair (*e.g.*, azimuth and elevation angles), a figure is given which shows the two angles, the coordinate system to which they are referred, and unit vectors in the directions of increases in the two angles. The unit vector in the direction of increasing elevation angle is used in calculating the refraction correction (the increase in the elevation angle due to atmospheric refraction). All of the unit vectors are used in calculating partial derivatives of the computed values of angular observables with respect to solve-for parameters. The formulation for these partial derivatives is given in Section 13.

Section 9.3 gives the formulation for computing angles (auxiliary angles or computed values of angular observables) of the spacecraft or a quasar at a tracking station on Earth. Angles can be computed on the down leg of the light path at the reception time t_3 or on the up leg of the light path at the transmission time t_1. The formulation for the unit vector **L** from the tracking station to the spacecraft or a quasar is given in Section 9.3.1. Section 9.3.2 gives the current and proposed formulation for calculating the refraction correction. Given the unit vector **L**, Section 9.3.3 gives the formulation for calculating angles at the reception time t_3 and at the transmission time t_1 at a tracking station on Earth.

Section 9.4 gives formulas for corrections to computed values of angular observables (measured at a tracking station on Earth) due to solve-for rotations of the reference coordinate system (to which the angle pair is referred) about each of its three axes.

Section 9.5 gives formulations for calculating auxiliary angles at an Earth satellite. Section 9.5.1 gives the formulation for calculating auxiliary angles at the reception time t_3 at the TOPEX satellite. Section 9.5.2 gives the formulation for calculating auxiliary angles at the transmission time t_2 at a GPS satellite.

9.2 COORDINATE SYSTEMS, ANGLES, AND UNIT VECTORS AT A TRACKING STATION ON EARTH

This section defines the angle pairs: hour angle (HA) and declination (δ) (Section 9.2.1), azimuth (σ) and elevation (γ) (Section 9.2.3), X and Y (Section 9.2.4), and X' and Y' (Section 9.2.5). Each angle pair is referred to an Earth-fixed rectangular coordinate system whose origin is located at the tracking station. The first of these angle pairs is referred to a coordinate system aligned with the true pole, prime meridian, and equator of date. The latter three angle pairs are referred to the north-east-zenith coordinate system, which is described in Section 9.2.2. For each angle pair, equations are given for the Earth-fixed components of unit vectors in the directions of increases in the two angles. Section 9.2.6 converts these unit vectors from Earth-fixed components to space-fixed components. The space-fixed unit vectors are used in Section 13 to calculate partial derivatives of computed values of angular observables with respect to solve-for parameters.

The antennas at the various tracking stations were not aligned very accurately when placed into the ground. However, they are calibrated occasionally (possibly a few times a year) so that the observed angles are referred to the true pole or true north direction of date (actually, the average direction during the calibration interval). The calibrated observed angles are accurate to about 0.001 degree at best. The estimated spherical or cylindrical coordinates of the tracking stations are referred to the mean pole, prime meridian, and equator of 1903.0. The station coordinates used to compute angles (auxiliary angles or computed values of angular observables) should be converted to values referred to the true pole, prime meridian, and equator of date using Eqs. (220), (228), and (231) to (233) of Moyer (1971). These equations are functions of the X and Y angular coordinates of the true pole of date relative to the mean pole of 1903.0 (see Section 5.2.5). Since X and Y are less than 0.0002 degrees, the station coordinates used to compute angles are not corrected for polar motion. Computed values of angular observables are corrected for atmospheric refraction. It will be seen in Section 9.3.2 that calculated refraction corrections are accurate to about 0.001 degree. From all of the above, it is seen that observed and computed values of angular observables and computed auxiliary angles have an accuracy on the order of 0.001 degree.

9.2.1 HOUR ANGLE AND DECLINATION

Figure 9–1 shows an Earth-fixed rectangular coordinate system whose origin is located at a tracking station on Earth. This x-y-z coordinate system is aligned with the Earth's true pole, prime (*i.e.*, 0°) meridian, and true equator of date. The z axis is parallel to the Earth's true axis of rotation, and the x axis is in the prime meridian.

The unit vector **L** is directed from the tracking station at the reception time t_3 or the transmission time t_1 to the spacecraft (a free spacecraft or a landed spacecraft on a celestial body) or a quasar. The angles $\lambda_{S/C}$ and δ are the east longitude and declination of the spacecraft or a quasar. The quantity λ is the east longitude of the tracking station. The observer's meridian contains the unit vectors **P** and **Q**. The unit vector **E** completes the observer's **QEP** rectangular coordinate system. The angle HA is the hour angle of the spacecraft or quasar.

Nominal computed values of observed hour angle HA and declination δ are based upon the geometry of Figure 9–1. However, the reference coordinate system **QEP** can be rotated through the small solve-for angles ζ' about **Q**, ε about **E**, and η' about **P**, thus changing the angle HA in the **QE** plane and the angle δ normal to it. Corrections to the computed values of HA and δ due to these small rotations are given in Section 9.4.

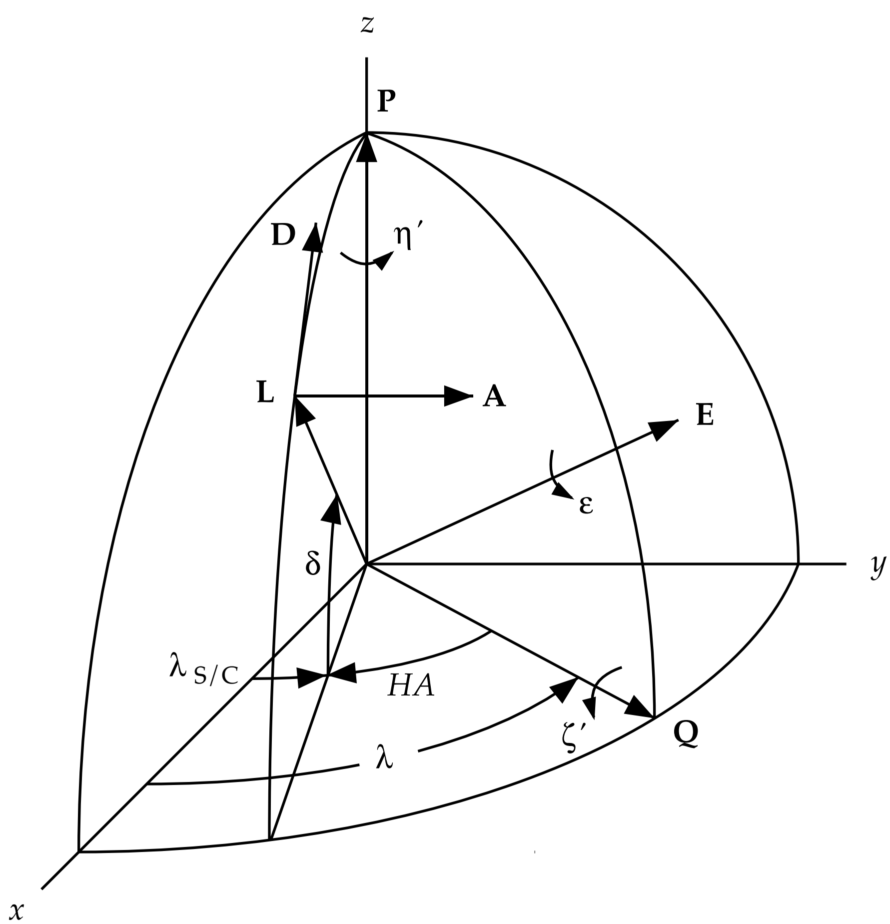

Figure 9–1 Hour Angle and Declination

The unit vectors **D** and **A** in the directions of increasing δ and $\lambda_{S/C}$ are used in computing the partial derivatives of δ and HA with respect to the solve-for parameters. The vector **A** is normal to **L** and **D**. The vectors **D** and **A** with rectangular components along the x, y, and z axes are given by:

$$\mathbf{D} = \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} -\sin\delta \cos\lambda_{\mathrm{S/C}} \\ -\sin\delta \sin\lambda_{\mathrm{S/C}} \\ \cos\delta \end{bmatrix} \tag{9–1}$$

$$\mathbf{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} -\sin\lambda_{\mathrm{S/C}} \\ \cos\lambda_{\mathrm{S/C}} \\ 0 \end{bmatrix} \tag{9–2}$$

9.2.2 THE NORTH-EAST-ZENITH COORDINATE SYSTEM

Figure 9–2 shows an Earth-fixed rectangular coordinate system whose origin is located at the center of the Earth. This x-y-z coordinate system is aligned with the Earth’s true pole, prime (*i.e.*, 0°) meridian, and true equator of date. The z axis is the Earth’s true axis of rotation and the x axis is in the prime meridian. The unit north **N**, east **E**, and zenith **Z** vectors originate at the tracking station S, which has an east longitude of λ. The unit vectors **N** and **Z** are in the tracking station’s meridian plane, and **E** is normal to it. The zenith vector **Z** makes an angle ϕ_g with the true equator of date, where ϕ_g is the geodetic latitude of the tracking station. The zenith vector **Z** is normal to the reference ellipsoid for the Earth.

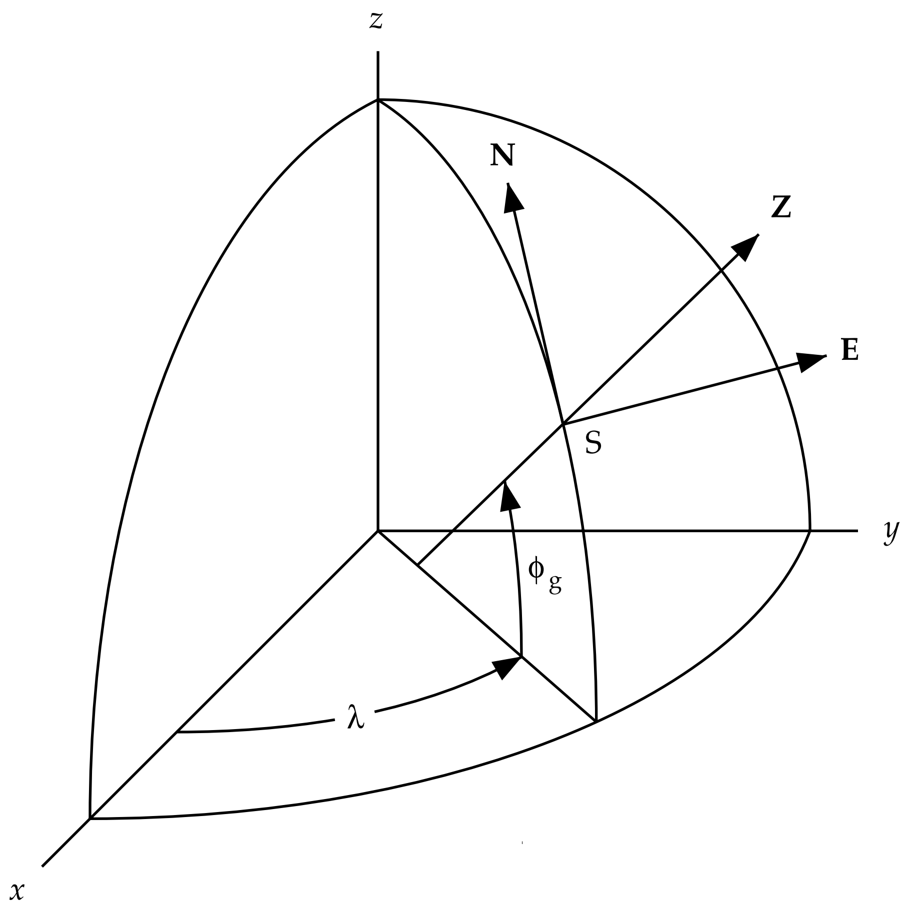

Figure 9–2 The North-East-Zenith Coordinate System

The angle pairs σ-γ, X-Y, and X'-Y' are referred to the rectangular **NEZ** coordinate system at the tracking station. The rectangular components of these unit vectors along the x, y, and z axes are:

$$\mathbf{N} = \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} -\sin\phi_g \cos\lambda \\ -\sin\phi_g \sin\lambda \\ \cos\phi_g \end{bmatrix} \tag{9–3}$$

$$\mathbf{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix} \tag{9–4}$$

$$\mathbf{Z} = \begin{bmatrix} Z_x \\ Z_y \\ Z_z \end{bmatrix} = \begin{bmatrix} \cos\phi_{\mathrm{g}} \cos\lambda \\ \cos\phi_{\mathrm{g}} \sin\lambda \\ \sin\phi_{\mathrm{g}} \end{bmatrix} \tag{9–5}$$

The geodetic latitude ϕ_{g} of the tracking station is computed from:

$$\phi_{\mathrm{g}} = \phi + \left(\phi_{\mathrm{g}} - \phi\right) \qquad \text{rad} \tag{9–6}$$

The spherical coordinates of the tracking station, which are referred to the true pole, prime meridian, and true equator of date are:

r = geocentric radius
ϕ = geocentric latitude
λ = east longitude

To sufficient accuracy, as discussed in Section 9.2, all of the equations in Section 9 are evaluated with the solve-for spherical coordinates r, ϕ, and λ of the tracking station, which are referred to the mean pole, prime meridian, and mean equator of 1903.0. In Eq. (9–6), the geodetic latitude ϕ_{g} minus the geocentric latitude ϕ can be calculated from:

$$\sin\left(\phi_{\mathrm{g}} - \phi\right) = \frac{e^2 a_{\mathrm{e}}}{r} \sin\phi \cos\phi \left[1 + \frac{e^2 a_e}{r} - e^2 \left(\frac{2a_e}{r} - \frac{1}{2}\right) \sin^2\phi\right] \tag{9–7}$$

where

e = eccentricity of reference ellipsoid for the Earth
a_{e} = mean equatorial radius of the Earth

The eccentricity e can be computed from the flattening f using:

$$e^2 = 2f - f^2 \tag{9–8}$$

From Chapter 1 of International Earth Rotation Service (1992) or Table 15.4 on page 700 of the *Explanatory Supplement* (1992),

$$a_e = 6378.136 \text{ km}$$
$$f = 1/298.257$$

9.2.3 AZIMUTH AND ELEVATION

Figure 9–3 shows the unit vector **L** directed from the tracking station S to the spacecraft or a quasar in the **NEZ** coordinate system centered at the tracking station. The angles σ and γ are the azimuth and elevation angles of the spacecraft or quasar. The **NEZ** reference coordinate system can be rotated through the small solve-for angles η about **N**, ε about **E**, and ζ about **Z**. Corrections to the computed values of σ and γ due to these small rotations are given in Section 9.4.

The unit vectors $\tilde{\mathbf{D}}$ and $\tilde{\mathbf{A}}$ (which are normal to **L**) in the directions of increasing γ and σ, respectively, with components along the axes of the Earth-fixed x-y-z rectangular coordinate system of Figure 9–2, which is aligned with the Earth's true pole, prime meridian, and true equator of date, are given by:

$$\tilde{\mathbf{D}} = \begin{bmatrix} \tilde{D}_x \\ \tilde{D}_y \\ \tilde{D}_z \end{bmatrix} = -\sin\gamma\cos\sigma\,\mathbf{N} - \sin\gamma\sin\sigma\,\mathbf{E} + \cos\gamma\,\mathbf{Z} \tag{9–9}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{A}_x \\ \tilde{A}_y \\ \tilde{A}_z \end{bmatrix} = -\sin\sigma\,\mathbf{N} + \cos\sigma\,\mathbf{E} \tag{9–10}$$

where **N**, **E**, and **Z** are given by Eqs. (9–3) to (9–5).

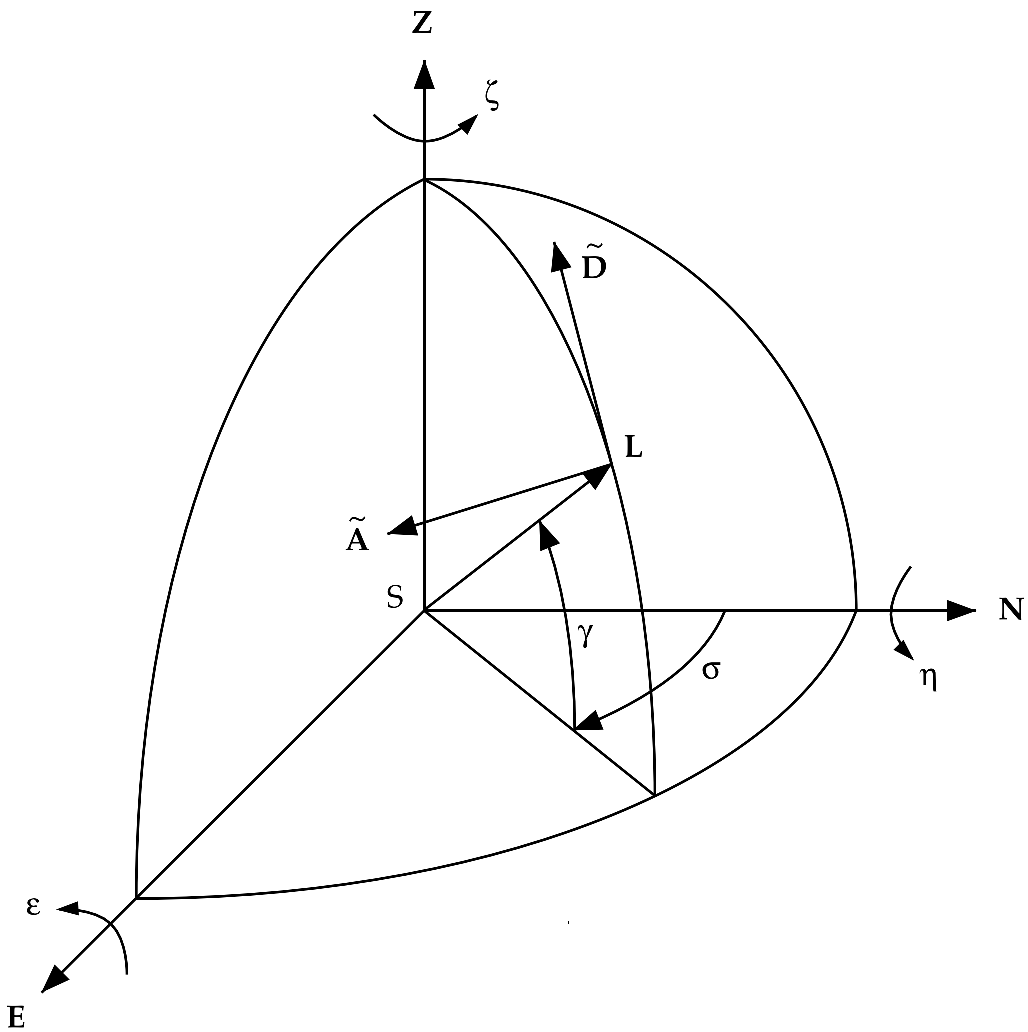

Figure 9–3 Azimuth and Elevation

9.2.4 *X* AND *Y* ANGLES

Figure 9–4 shows the unit vector **L** directed from the tracking station S to the spacecraft or a quasar in the **NEZ** coordinate system centered at the tracking station. The X and Y angles of the spacecraft or a quasar are shown. The **NEZ** reference coordinate system can be rotated through the small solve-for angles η about **N**, ε about **E**, and ζ about **Z**. Corrections to the computed values of X and Y due to these small rotations are given in Section 9.4.

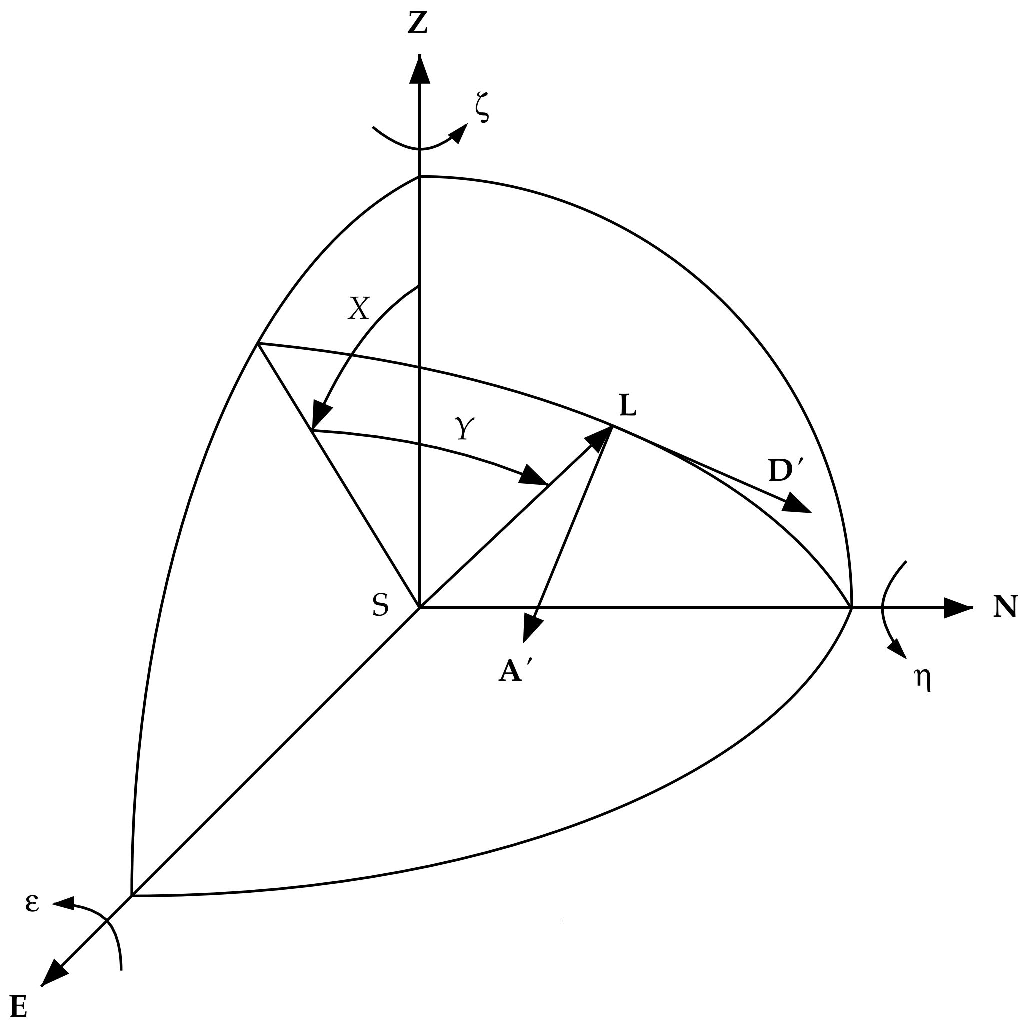

Figure 9–4 X and Y Angles

The unit vectors **D′** and **A′** (which are normal to **L**) in the directions of increasing Y and X, respectively, with components along the axes of the Earth-fixed x-y-z rectangular coordinate system of Figure 9–2, which is aligned with the Earth's true pole, prime meridian, and true equator of date, are given by:

$$\mathbf{D'} = \begin{bmatrix} D'_x \\ D'_y \\ D'_z \end{bmatrix} = \cos Y\,\mathbf{N} - \sin Y \sin X\,\mathbf{E} - \sin Y \cos X\,\mathbf{Z} \qquad (9\text{–}11)$$

$$\mathbf{A}' = \begin{bmatrix} A'_x \\ A'_y \\ A'_z \end{bmatrix} = \cos X\,\mathbf{E} - \sin X\,\mathbf{Z} \tag{9–12}$$

where **N**, **E**, and **Z** are given by Eqs. (9–3) to (9–5).

9.2.5 X' AND Y' ANGLES

Figure 9–5 shows the unit vector **L** directed from the tracking station S to the spacecraft or a quasar in the **NEZ** coordinate system centered at the tracking station. Note that Figure 9–5 shows the unit south **S** vector instead of the unit north **N** vector, where **S** = – **N**. The X' and Y' angles of the spacecraft or a quasar are shown. The **NEZ** reference coordinate system can be rotated through the small solve-for angles η about **N** (shown as – η about **S**), ε about **E**, and ζ about **Z**. Corrections to the computed values of X' and Y' due to these small rotations are given in Section 9.4.

The unit vectors **D″** and **A″** (which are normal to **L**) in the directions of increasing Y' and X', respectively, with components along the axes of the Earth-fixed *x*-*y*-*z* rectangular coordinate system of Figure 9–2, which is aligned with the Earth's true pole, prime meridian, and true equator of date, are given by:

$$\mathbf{D}'' = \begin{bmatrix} D''_x \\ D''_y \\ D''_z \end{bmatrix} = \sin Y' \sin X'\,\mathbf{N} + \cos Y'\,\mathbf{E} - \sin Y' \cos X'\,\mathbf{Z} \tag{9–13}$$

$$\mathbf{A}'' = \begin{bmatrix} A''_x \\ A''_y \\ A''_z \end{bmatrix} = -\cos X'\,\mathbf{N} - \sin X'\,\mathbf{Z} \tag{9–14}$$

where **N**, **E**, and **Z** are given by Eqs. (9–3) to (9–5).

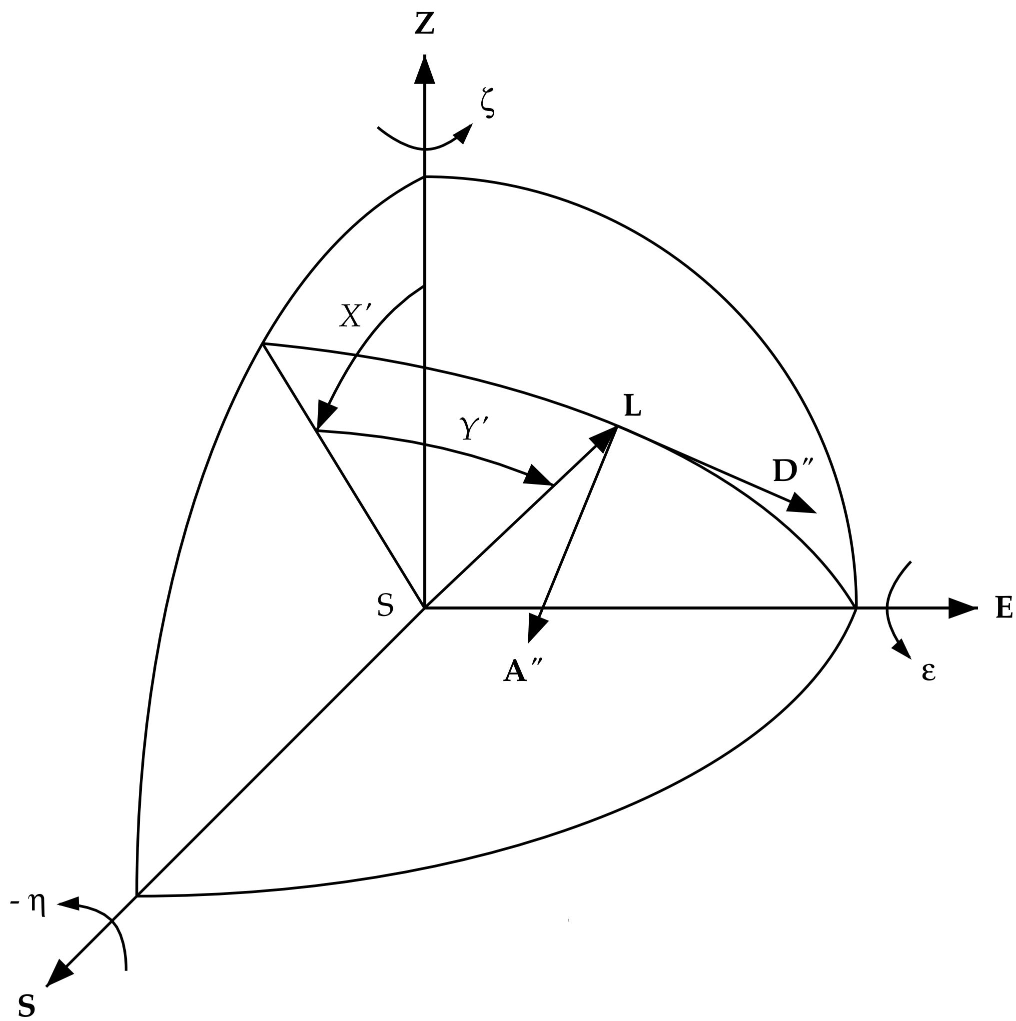

Figure 9–5 X' **and** Y' **Angles**

9.2.6 CONVERSION OF EARTH-FIXED UNIT VECTORS TO SPACE-FIXED UNIT VECTORS

In order to calculate partial derivatives of computed values of angular observables with respect to the solve-for parameter vector $\mathbf{q}$ in Section 13, the unit vectors $\mathbf{D}$, $\mathbf{A}$, $\tilde{\mathbf{D}}$, $\tilde{\mathbf{A}}$, $\mathbf{D}'$, $\mathbf{A}'$, $\mathbf{D}''$, and $\mathbf{A}''$ calculated at the reception time t_3 at the tracking station on Earth must be transformed from Earth-fixed rectangular components to the space-fixed rectangular components (subscript

SF) of the celestial reference frame of the planetary ephemeris (see Section 3.1.1). From Eq. (5–113),

$$\mathbf{D}_{\mathrm{SF}} = T_{\mathrm{E}}\left(t_3\right)\mathbf{D} \qquad \mathbf{D} \rightarrow \mathbf{A},\ \tilde{\mathbf{D}},\ \tilde{\mathbf{A}},\ \mathbf{D}',\ \mathbf{A}',\ \mathbf{D}'',\ \mathbf{A}'' \qquad (9\text{–}15)$$

where the Earth-fixed to space-fixed transformation matrix $T_{\mathrm{E}}(t_3)$ at the reception time t_3 at the tracking station on Earth is calculated from the formulation of Section 5.3. It is available from Step 2 of the spacecraft light-time solution (Section 8.3.6).

The unit vectors $\mathbf{D}, \mathbf{A}, \tilde{\mathbf{D}}, \tilde{\mathbf{A}}, \mathbf{D}', \mathbf{A}', \mathbf{D}''$, and $\mathbf{A}''$, with Earth-fixed rectangular components referred to the Earth's true pole, prime meridian, and true equator of date, are calculated from Eqs. (9–1), (9–2), and (9–9) to (9–14). In these equations, the **N**, **E**, and **Z** vectors are evaluated using Eqs. (9–3) to (9–8). Calculation of the unit vectors $\mathbf{D}, \mathbf{A}, \tilde{\mathbf{D}}, \tilde{\mathbf{A}}, \mathbf{D}', \mathbf{A}', \mathbf{D}''$, and $\mathbf{A}''$ requires computed values of the east longitude $\lambda_{\mathrm{S/C}}$ and declination δ of the spacecraft, the azimuth σ and elevation γ of the spacecraft, the X and Y angles of the spacecraft, and the X' and Y' angles of the spacecraft, respectively, at the reception time t_3 at the tracking station on Earth. These angles are calculated from the formulation given in Section 9.3.

9.3 COMPUTATION OF ANGLES AT RECEPTION AND TRANSMISSION TIMES AT A TRACKING STATION ON EARTH

9.3.1 UNIT VECTOR L

The unit vector **L** is directed from a receiving or transmitting station on Earth toward the spacecraft (a free spacecraft or a landed spacecraft on a celestial body) or from a receiving station on Earth toward a quasar.

The unit vector $\mathbf{L}_{\mathrm{SF}}$ directed from the receiving station on Earth at the reception time t_3 toward the spacecraft at the reflection time or transmission time t_2, with rectangular components referred to the space-fixed (SF) coordinate system (mean Earth equator and equinox of J2000) of the planetary ephemeris (see Section 3.1.1) is given by:

$$\mathbf{L}_{SF} = -\frac{\mathbf{r}_{23}}{r_{23}} \tag{9–16}$$

The down-leg unit vector $\mathbf{r}_{23}/r_{23}$ is calculated from Eqs. (8–56) to (8–58) in the Solar-System barycentric frame of reference. In the local geocentric frame of reference, the superscript C in these equations for the Solar-System barycenter becomes E for the Earth. The space-fixed unit vector $\mathbf{L}_{SF}$ directed from the transmitting station on Earth at the transmission time t_1 toward the spacecraft at t_2 is given by:

$$\mathbf{L}_{SF} = \frac{\mathbf{r}_{12}}{r_{12}} \tag{9–17}$$

where the up-leg unit vector $\mathbf{r}_{12}/r_{12}$ is also calculated from Eqs. (8–56) to (8–58). For narrowband or wideband quasar interferometric data types, the space-fixed unit vector $\mathbf{L}_{SF}$ directed from receiving station 1 or 2 on Earth toward a quasar is given by:

$$\mathbf{L}_{SF} = \mathbf{L}_{Q} \tag{9–18}$$

where the unit vector $\mathbf{L}_Q$ toward the quasar is calculated from Eqs. (8–92) and (8–93).

The velocity vector of light on the up leg of the light path in the Solar-System barycentric or local geocentric frame of reference is $c\mathbf{L}_{SF}$, where c is the speed of light and $\mathbf{L}_{SF}$ is given by Eq. (9–17). On the down leg of the light path, the velocity vector of light is $-c\mathbf{L}_{SF}$, where $\mathbf{L}_{SF}$ is given by Eq. (9–16) or (9–18). The velocity vector relative to the transmitting station on the up leg is given by $c\mathbf{L}_{SF} - \dot{\mathbf{r}}_1^C(t_1)$, where $\dot{\mathbf{r}}_1^C(t_1)$ is the velocity vector of the transmitting station on Earth at the transmission time t_1 relative to the Solar-System barycenter C (the Earth E in the local geocentric frame of reference). The velocity vector relative to the receiving station on the down leg is given by $-c\mathbf{L}_{\mathrm{SF}} - \dot{\mathbf{r}}_3^{\mathrm{C}}(t_3)$, where $\dot{\mathbf{r}}_3^{\mathrm{C}}(t_3)$ is the velocity vector of the receiving station on Earth at the reception time t_3. For narrowband or wideband quasar interferometric data types, the velocity vectors of receiving stations 1 and 2 on Earth at their corresponding reception times t_1 and t_2 are denoted as $\dot{\mathbf{r}}_1^{\mathrm{C}}(t_1)$ and $\dot{\mathbf{r}}_2^{\mathrm{C}}(t_2)$, respectively. Let $\mathbf{L}_{\mathrm{SF}} + \Delta\mathbf{L}$ denote the unit vector from the tracking station on Earth to the spacecraft or a quasar which is aligned with the velocity vector of light relative to the transmitting station on the up leg or the negative of the velocity vector of light relative to the receiving station on the down leg. The correction vector $\Delta\mathbf{L}$ is the stellar aberration of light due to the velocity of the transmitter or the receiver. From the above equations, the aberration correction $\Delta\mathbf{L}$ for the down leg of the light path is given by:

$$\Delta\mathbf{L} = \frac{\dot{\mathbf{r}}_3^{\mathrm{C}}(t_3)}{c} \tag{9–19}$$

where, as noted above, the subscripts 3 become 1 and 2 for reception at receiving stations 1 and 2 on Earth for quasar interferometric data types. When calculating in the local geocentric space-time frame of reference, the numerator of Eq. (9–19) changes from the Solar-System barycentric velocity vector of the receiver to the geocentric velocity vector of the receiver. The aberration correction $\Delta\mathbf{L}$ for the up leg of the light path is given by:

$$\Delta\mathbf{L} = -\frac{\dot{\mathbf{r}}_1^{\mathrm{C}}(t_1)}{c} \tag{9–20}$$

Given $\mathbf{L}_{\mathrm{SF}}$ for the down leg of the light path calculated from Eq. (9–16) or Eq. (9–18) and the aberration correction $\Delta\mathbf{L}$ calculated from Eq. (9–19), the space-fixed unit vector from the receiving station on Earth at the reception time to the spacecraft or a quasar, which is corrected for stellar aberration, is given by:

$$\mathbf{L}_{\mathrm{SFA}} = \frac{\mathbf{L}_{\mathrm{SF}} + \Delta\mathbf{L}}{\left|\mathbf{L}_{\mathrm{SF}} + \Delta\mathbf{L}\right|} \tag{9–21}$$

where the denominator is the magnitude of the vector in the numerator. Given $\mathbf{L}_{SF}$ for the up leg of the light path calculated from Eq. (9–17) and the aberration correction $\Delta\mathbf{L}$ calculated from Eq. (9–20), the space-fixed unit vector from the transmitting station on Earth at t_1 to the spacecraft at t_2, which is corrected for stellar aberration, is given by Eq. (9–21).

Equations (9–19) and (9–20) for the stellar aberration of light were derived from Newtonian theory. More accurate expressions can be derived from the Lorentz transformation of special relativity. The first-order terms from special relativity and Newtonian theory are the same. The second-order term from special relativity differs from the corresponding term of Newtonian theory by less than 2 x 10^{-7} degree, which is negligible compared to the previously stated accuracy of 0.001 degree for observed and computed angles.

The unit vector $\mathbf{L}_{SFA}$ from the transmitting or receiving station on Earth to the spacecraft or a quasar, calculated from Eqs. (9–16) to (9–21), can be transformed from space-fixed to Earth-fixed components by using the transpose of Eq. (5–113):

$$\mathbf{L}_{EF} = T_E(t_3)^T \ \mathbf{L}_{SFA} \qquad 3 \to 1, 2 \qquad \text{(9–22)}$$

where the Earth-fixed rectangular components of $\mathbf{L}_{EF}$ are referred to the Earth's true pole, prime meridian, and true equator of date. For the down leg of the light path, the Earth-fixed to space-fixed transformation matrix T_E is calculated at the reception time t_3 at the receiving station on Earth. For quasar interferometric data types, the reception times at receiving stations 1 and 2 on Earth are denoted as t_1 and t_2, respectively. For the up leg of the light path, T_E is calculated at the transmission time t_1 at the transmitting station on Earth.

The unit vector $\mathbf{L}_{EF}$ does not account for the bending of the raypath due to atmospheric refraction, which increases the elevation angle γ of the raypath by $\Delta_r\gamma$. The existing formulation and the proposed formulation for calculating the refraction correction $\Delta_r\gamma$ are given in Section 9.3.2. The refraction correction is a function of the elevation angle γ and atmospheric parameters. Computed values of angular observables are corrected for refraction. If program Regres calculates partial derivatives (*i.e.*, it is a fit case), the calculated auxiliary angles are not corrected for refraction. However, if partial derivatives are not being calculated (*i.e.*, tracking data is not being fit to), the user may request that refraction corrections be added to auxiliary angles. This is done if auxiliary angles are used as antenna pointing predictions.

The argument for the tropospheric correction, which is the delay of the radio signal due to the troposphere, is the unrefracted elevation angle γ. Antenna corrections, which are non-zero if the two axes of the antenna at the tracking station do not intersect, are described in Section 10. They account for the light time from the “station location”, which is on the primary axis of the antenna, to the tracking point, which is on the secondary axis of the antenna. The antenna corrections are calculated from the antenna angles shown in Figure 9–1 and Figures 9–3 to 9–5. The errors due to calculating antenna corrections from unrefracted auxiliary angles instead of refracted angles are negligible.

Referring to Figure 9–3, the change in $\mathbf{L}_{\mathrm{EF}}$ due to atmospheric refraction is $\tan\Delta_{\mathrm{r}}\gamma\,\tilde{\mathbf{D}}$. Hence, the unit vector $\mathbf{L}_{\mathrm{EFR}}$, which is the unit vector $\mathbf{L}_{\mathrm{EF}}$ corrected for atmospheric refraction, is given by:

$$\mathbf{L}_{\mathrm{EFR}} = \frac{\mathbf{L}_{\mathrm{EF}} + \tan\Delta_{\mathrm{r}}\gamma\,\tilde{\mathbf{D}}}{\left|\mathbf{L}_{\mathrm{EF}} + \tan\Delta_{\mathrm{r}}\gamma\,\tilde{\mathbf{D}}\right|} \tag{9–23}$$

where $\tilde{\mathbf{D}}$ is calculated from Eq. (9–9). Calculation of $\Delta_{\mathrm{r}}\gamma$ and $\tilde{\mathbf{D}}$ requires the azimuth σ and elevation γ angles of the spacecraft or quasar. Approximate values are obtained from Eqs. (9–42) to (9–44) of Section 9.3.3.2, evaluated with $\mathbf{L}_{\mathrm{EF}}$ given by Eq. (9–22) instead of $\mathbf{L}_{\mathrm{EFR}}$.

All quantities required to evaluate Eqs. (9–16) to (9–22) are available from the spacecraft light-time solution (Section 8.3.6) or the quasar light-time solution (Section 8.4.3).

9.3.2 REFRACTION CORRECTION $\Delta_r \gamma$

The refraction correction $\Delta_r \gamma$ is the increase in the elevation angle due to atmospheric refraction. Subsections 9.3.2.1 and 9.3.2.2 give the existing model and the proposed model for calculating the refraction correction $\Delta_r \gamma$. The existing model is a modified form of the Berman-Rockwell model. The proposed model is due to Lanyi.

9.3.2.1 Modified Berman-Rockwell Model

The unmodified Berman-Rockwell model for the refraction correction $\Delta_r \gamma$ is given in Section III.D on page 12 of Berman and Rockwell (1975) and in Berman (1977). This model is an empirical fit to atmospheric data. The model contains pressure, temperature, and relative humidity factors. One of the modifications to the original Berman-Rockwell model was to delete the relative humidity factor, which means that the modified model applies for a dry atmosphere. In the temperature factor, the surface temperature was set to 284.5 K. In the pressure factor, the surface pressure was replaced with 2.75 times the surface refractivity. Note that the index of refraction of the atmosphere is unity plus one millionth of the refractivity. In the modified model, each tracking station has its own yearly average value of the surface refractivity. Also, the pressure and temperature factors each contain the same coding error. This error is probably negligible except at very low elevation angles.

The sources of the modifications to the original Berman-Rockwell model are currently unknown. Also, the surface refractivity versus tracking station table needs to be greatly expanded since a large number of tracking stations have been created since the model was implemented. The Berman-Rockwell model for the atmospheric refraction correction $\Delta_r \gamma$ is a function of the unrefracted elevation angle γ and atmospheric parameters, which are included in the model and the accompanying table of surface refractivities versus tracking station number.

I suggest that we replace the Berman-Rockwell model for atmospheric refraction with the more-accurate Lanyi model. The only significant error in the refraction correction $\Delta_r \gamma$ calculated from the Lanyi model is due to errors in the input atmospheric parameters. I suggest that we evaluate the Lanyi model with monthly average values of atmospheric parameters at each DSN complex. The ODP user will have the option of overstoring the average atmospheric parameters for the current month with near-real-time measured values.

9.3.2.2 Lanyi Model

The Lanyi model for the refraction correction $\Delta_r \gamma$ is given in Lanyi (1989). The model consists of Eqs. (2) to (18), which are in Section III. Since there are some units which must be added to these equations, I have included the whole set of equations in Subsection 9.3.2.2.1. Subsection 9.3.2.2.2 discusses how the atmospheric parameters can be obtained and used in evaluating the Lanyi model.

9.3.2.2.1 Equations

The inputs to the Lanyi model for the refraction correction $\Delta_r \gamma$ are the unrefracted elevation angle γ of the spacecraft or a quasar and the following three atmospheric parameters:

p_0 = total surface pressure, mbar. The nominal value is 1013.25 mbar (101,325 Pa) at mean sea level.

T_0 = surface temperature, Kelvins. The mean DSN value is 292 K.

RH_0 = surface relative humidity, expressed as a fraction between 0 and 1.

The third atmospheric parameter in the Lanyi model can be the dew point temperature $T_{0C\ dew}$ or the surface relative humidity RH_0, which are related by formula. Since the existing tables which contain monthly average values of atmospheric parameters contain the relative humidity, I have used it as the third atmospheric parameter in the Lanyi model.

The following algorithm can be used to calculate the refraction correction $\Delta_r \gamma$ from the model of Lanyi:

1. Calculate the mean height of dry air h_d and the mean height of water vapor h_w:

$$h_d = 0.86 \times 8.567 \left(T_0 / 292\right) \times 10^3 \qquad \text{m} \qquad (9\text{–}24)$$

$$h_w = 2.4 \times 10^3 \qquad \text{m} \qquad (9\text{–}25)$$

2. Calculate the water vapor surface pressure p_{0w} and the dry surface pressure p_{0d}:

$$p_{0w} = 6.11\, RH_0\, e^{\left(\frac{17.27\, T_{0C}}{237.3 + T_{0C}}\right)} \qquad \text{mbar} \qquad (9\text{–}26)$$

where

$$T_{0C} = T_0 - 273.16 \qquad (9\text{–}27)$$

$$p_{0d} = p_0 - p_{0w} \qquad \text{mbar} \qquad (9\text{–}28)$$

3. Calculate the dry surface refractivity χ_{0d}, the water vapor surface refractivity χ_{0w}, and the total surface refractivity χ_0:

$$\chi_{0d} = \left(77.6 \times 10^{-6}\right) p_{0d} \,/\, T_0 \qquad (9\text{–}29)$$

$$\chi_{0w} = \left(377.6 \times 10^3 \,/\, T_0 + 64.8\right) \times 10^{-6}\, p_{0w} \,/\, T_0 \qquad (9\text{–}30)$$

$$\chi_0 = \chi_{0d} + \chi_{0w} \qquad (9\text{–}31)$$

4. Calculate the dry zenith delay Z_{dry} and the wet zenith delay Z_{wet}:

$$Z_{dry} = 0.22768\, p_{0d} \times 10^{-2} \qquad \text{m} \qquad (9\text{–}32)$$

$$Z_{\mathrm{wet}} = \chi_{0\mathrm{w}}\, h_{\mathrm{w}} \qquad \mathrm{m} \qquad (9\text{–}33)$$

5. Calculate the function $a(\gamma)$, where γ is the unrefracted elevation angle of the spacecraft or a quasar:

$$a(\gamma) = \left\{ \frac{Z_{\mathrm{dry}}}{\left[1 - \left(\dfrac{\cos\gamma}{1+(h_{\mathrm{d}}/R)}\right)^2\right]^{\frac{3}{2}}} + \frac{Z_{\mathrm{wet}}}{\left[1 - \left(\dfrac{\cos\gamma}{1+(h_{\mathrm{w}}/R)}\right)^2\right]^{\frac{3}{2}}} \right\} \frac{\sin\gamma}{R} \qquad (9\text{–}34)$$

where R is the mean radius of curvature of the Earth in meters = 6.378 x 10^6 m.

6. The function $F(x)$ is defined to be:

$$F(x) = \frac{1}{1 + \frac{1}{2}\left(\sqrt{1+2x} - 1\right)} \qquad (9\text{–}35)$$

7. Calculate the refraction correction $\Delta_{\mathrm{r}}\gamma$:

$$\Delta_{\mathrm{r}}\gamma = \left[\frac{\chi_0 - a(\gamma)}{\tan\gamma}\right] F\left[\frac{\chi_0 - a(\gamma)}{\tan^2\gamma}\right] \qquad \mathrm{rad} \qquad (9\text{–}36)$$

where $F(x)$ is evaluated from Eq. (9–35).

9.3.2.2.2 Atmospheric Parameters

The following changes to the ODP will provide the surface atmospheric parameters needed to calculate the refraction correction $\Delta_{\mathrm{r}}\gamma$ from the model of Lanyi. For each of the three DSN complexes (Goldstone, Madrid, and Canberra), add a table to the GIN file which contains monthly average values of the total surface pressure p_0 in mbar, the surface temperature T_0 in Kelvins, and the surface relative humidity RH_0, which is a fraction that varies from 0 to 1. Also, we need separate constant inputs for each of these three variables, which will be used for stations that are not at one of the three major complexes.

Angular observables are "fit to" mainly during the first half hour or so of the spacecraft trajectory after booster burnout, when the spacecraft is very near the Earth. If it is desired to calculate refraction corrections from near-current values of surface atmospheric parameters instead of the monthly average values obtained from the above-mentioned tables, the data in the tables for the time period of the tracking data can be overstored with near-current values, which are available.

The DSN has been measuring the surface atmospheric parameters every minute at one location at each of the three major complexes for many years. The VLBI group obtains 30-minute averages of these atmospheric parameters, which can be obtained from readily-available files. This data can be used to overstore the monthly average values on the GIN file when more accurate refraction corrections are desired.

Tables 7, 8, and 10 of Chao (1974) give monthly average values of surface pressure p_0, surface temperature T_0, and relative humidity RH_0 at the Goldstone, Madrid, and Canberra complexes. The data in these tables can be used as the nominal values in the corresponding tables on the GIN file. However, the surface temperature data will have to be converted from °C to K by adding 273.16, and the relative humidity data will have to be converted from a percentage to a fraction by moving the decimal point two places to the left. The data in these three tables was measured in 1967 and 1968. It should be replaced with averages from DSN data obtained during the last few years.

Each of the three atmospheric tables will be used for all of the tracking stations at the corresponding complex. Ignoring the variations in the pressure and temperature between the stations in a complex can result in errors in the calculated refraction correction $\Delta_r \gamma$ of up to about 2%. The errors in $\Delta_r \gamma$ can be up to about 0.003 degrees at an elevation angle γ of 6 degrees and 0.005 degrees at 3 degrees.

9.3.3 COMPUTED ANGLES

Computed angles are calculated from the unrefracted Earth-fixed unit vector $\mathbf{L}_{\mathrm{EF}}$ from a tracking station on Earth to the spacecraft or a quasar given by Eq. (9–22) or the corresponding refracted unit vector $\mathbf{L}_{\mathrm{EFR}}$ given by Eq. (9–23). In this section, we will denote either of these unit vectors as $\mathbf{L}$:

$$\mathbf{L} = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} \tag{9–37}$$

where the Earth-fixed rectangular components of $\mathbf{L}$ are referred to the Earth's true pole, prime meridian, and true equator of date. Computed values of angular observables are calculated from $\mathbf{L}_{\mathrm{EFR}}$. For all fit cases, computed auxiliary angles are calculated from $\mathbf{L}_{\mathrm{EF}}$. For non-fit cases, computed auxiliary angles can be calculated from the unrefracted or the refracted unit vector, as specified by the ODP user. The computed values of angular observables are in units of degrees. The computed auxiliary angles are in units of radians.

Calculation of all angles except hour angle HA and declination δ requires the unit north $\mathbf{N}$, east $\mathbf{E}$, and zenith $\mathbf{Z}$ vectors with rectangular components referred to the true pole, prime meridian, and equator of date. They are computed from Eqs. (9–3) to (9–5).

9.3.3.1 Hour Angle and Declination

From Figure 9–1, the declination δ of the spacecraft or a quasar, which varies from $-90°$ to $90°$, can be calculated from:

$$\sin\delta = L_z \tag{9–38}$$

The east longitude $\lambda_{S/C}$ of the spacecraft or a quasar, which varies from $0°$ to $360°$, can be calculated from:

$$\sin \lambda_{S/C} = \frac{L_y}{\cos \delta} \tag{9–39}$$

$$\cos \lambda_{S/C} = \frac{L_x}{\cos \delta} \tag{9–40}$$

Calculate the hour angle HA of the spacecraft or a quasar from:

$$HA = \lambda - \lambda_{S/C} \tag{9–41}$$

where λ is the east longitude of the tracking station, referred to the true pole, prime meridian, and equator of date. When HA is the computed value of an angular observable and is negative, add $360°$ to HA so it will be between $0°$ and $360°$.

9.3.3.2 Azimuth and Elevation

From Figure 9–3, the elevation angle γ of the spacecraft or a quasar, which varies from $0°$ to $90°$, can be calculated from:

$$\sin \gamma = \mathbf{L} \cdot \mathbf{Z} \tag{9–42}$$

The azimuth angle σ of the spacecraft or a quasar, which varies from $0°$ to $360°$, can be calculated from:

$$\sin \sigma = \frac{\mathbf{L} \cdot \mathbf{E}}{\cos \gamma} \tag{9–43}$$

$$\cos \sigma = \frac{\mathbf{L} \cdot \mathbf{N}}{\cos \gamma} \tag{9–44}$$

Note that σ is indeterminate for $\gamma = 90°$.

9.3.3.3 X and Y Angles

From Figure 9–4, the angle Y, which varies from $-90°$ to $90°$, can be calculated from:

$$\sin Y = \mathbf{L} \cdot \mathbf{N} \tag{9–45}$$

The angle X, which varies from $-90°$ to $90°$, can be calculated from:

$$\sin X = \frac{\mathbf{L} \cdot \mathbf{E}}{\cos Y} \tag{9–46}$$

Note that X is indeterminate when $Y = \pm 90°$, which can only occur when the spacecraft or a quasar is on the horizon.

9.3.3.4 X' and Y' Angles

From Figure 9–5, the angle Y', which varies from $-90°$ to $90°$, can be calculated from:

$$\sin Y' = \mathbf{L} \cdot \mathbf{E} \tag{9–47}$$

The angle X', which varies from $-90°$ to $90°$, can be calculated from:

$$\sin X' = -\frac{\mathbf{L} \cdot \mathbf{N}}{\cos Y'} \tag{9–48}$$

where **S** in Figure 9–5 is equal to –**N**. Note that X' is indeterminate when $Y' = \pm 90°$, which can only occur when the spacecraft or a quasar is on the horizon.

9.4 CORRECTIONS DUE TO SMALL ROTATIONS OF REFERENCE COORDINATE SYSTEM AT TRACKING STATION ON EARTH

This section gives equations for differential corrections to the computed values of angular observables due to the small solve-for rotations of the reference coordinate system at the tracking station about each of its three mutually perpendicular axes.

From Figure 9–1, the hour angle HA and declination δ of the spacecraft or a quasar are referred to the **QEP** rectangular coordinate system at the tracking station. Eqs. (9–38) to (9–41) for calculating the computed values of HA and δ observables are not explicit functions of the **Q**, **E**, and **P** unit vectors. The desired equations, which are needed in this section, are:

$$\sin\delta = \mathbf{L}\cdot\mathbf{P} \tag{9–49}$$

$$\cos\delta \sin HA = -\,\mathbf{L}\cdot\mathbf{E} \tag{9–50}$$

$$\cos\delta \cos HA = \mathbf{L}\cdot\mathbf{Q} \tag{9–51}$$

The variations in **Q**, **E**, and **P** due to rotating the reference coordinate system **QEP** through the small solve-for angles ζ' about **Q**, ε about **E**, and η' about **P** are given by:

$$\Delta\mathbf{Q} = \eta'\,\mathbf{E} - \varepsilon\,\mathbf{P} \tag{9–52}$$

$$\Delta\mathbf{E} = \zeta'\,\mathbf{P} - \eta'\,\mathbf{Q} \tag{9–53}$$

$$\Delta\mathbf{P} = \varepsilon\,\mathbf{Q} - \zeta'\,\mathbf{E} \tag{9–54}$$

Differentiating Eqs. (9–49) to (9–51) and substituting Eqs. (9–52) to (9–54) gives the following equations for the differential corrections to the computed values of HA and δ observables due to the small solve-for rotations ζ', ε, and η':

$$\Delta\delta = \zeta' \sin HA + \varepsilon \cos HA \qquad \text{deg} \qquad (9\text{–}55)$$

$$\Delta HA = \eta' + \tan\delta \left(\varepsilon \sin HA - \zeta' \cos HA\right) \qquad \text{deg} \qquad (9\text{–}56)$$

Eqs. (9–42) to (9–48) for the computed values of azimuth σ, elevation γ, X, Y, X', and Y' observables are explicit functions of the unit north **N**, east **E**, and zenith **Z** vectors. From Figure 9–3, the variations in **N**, **E**, and **Z** due to rotating the reference coordinate system **NEZ** through the small solve-for rotations η about **N**, ε about **E**, and ζ about **Z** are given by:

$$\Delta\mathbf{N} = \varepsilon\,\mathbf{Z} - \zeta\,\mathbf{E} \qquad (9\text{–}57)$$

$$\Delta\mathbf{E} = \zeta\,\mathbf{N} - \eta\,\mathbf{Z} \qquad (9\text{–}58)$$

$$\Delta\mathbf{Z} = \eta\,\mathbf{E} - \varepsilon\,\mathbf{N} \qquad (9\text{–}59)$$

Differentiating Eqs. (9–42) to (9–48) and substituting Eqs. (9–57) to (9–59) gives the following equations for the differential corrections to the computed values of azimuth σ, elevation γ, X, Y, X', and Y' observables due to the small solve-for rotations η, ε, and ζ. For azimuth σ and elevation γ:

$$\Delta\gamma = \eta \sin\sigma - \varepsilon \cos\sigma \qquad \text{deg} \qquad (9\text{–}60)$$

$$\Delta\sigma = \zeta - \tan\gamma \left(\eta \cos\sigma + \varepsilon \sin\sigma\right) \qquad \text{deg} \qquad (9\text{–}61)$$

For the angles X and Y:

$$\Delta Y = -\,\zeta \sin X + \varepsilon \cos X \qquad \text{deg} \qquad (9\text{–}62)$$

$$\Delta X = -\,\eta + \tan Y \left(\varepsilon \sin X + \zeta \cos X\right) \qquad \text{deg} \qquad (9\text{–}63)$$

For the angles X' and Y':

$$\Delta Y' = -\,\zeta \sin X' - \eta \cos X' \qquad \text{deg} \qquad (9\text{–}64)$$

$$\Delta X' = -\varepsilon + \tan Y' \left(\zeta \cos X' - \eta \sin X' \right) \qquad \text{deg} \qquad (9\text{–}65)$$

9.5 COMPUTATION OF AUXILIARY ANGLES AT EARTH SATELLITES

9.5.1 AUXILIARY ANGLES AT RECEPTION TIME AT TOPEX SATELLITE

Let **X**, **Y**, and **Z** be unit vectors aligned with the x, y, and z axes of the spacecraft-fixed right-handed rectangular coordinate system of the TOPEX satellite, directed outward from the origin of the coordinate system. Interpolation of the PV file for the TOPEX satellite at the reception time t_3(ET) at the TOPEX satellite gives the space-fixed rectangular components of the unit vectors **X**, **Y**, and **Z** referred to the mean Earth equator and equinox of J2000 (specifically, the space-fixed rectangular coordinate system of the planetary ephemeris). This interpolation is performed in Step 2 of the spacecraft light-time solution (see Section 8.3.6, Step 2 and Section 7.3.3, Step 3). The unit vector **Z** for the TOPEX satellite is perpendicular to the reference ellipsoid for the Earth, directed down. Normally, the **X** axis is aligned with the velocity vector of the TOPEX satellite.

The space-fixed unit vector **L** directed from the TOPEX satellite at the reception time t_3 to the transmitting GPS satellite at the transmission time t_2 can be calculated from Eqs. (9–16), (9–19), and (9–21). In Eq. (9–19) for the correction due to stellar aberration, $\dot{\mathbf{r}}_3^C(t_3)$ is the space-fixed velocity vector of the TOPEX satellite relative to the Solar-System barycenter C in that frame of reference and relative to the Earth E in the local geocentric space-time frame of reference.

The auxiliary angles computed at the TOPEX satellite are the azimuth σ and elevation γ angles. The azimuth angle σ is measured in the x-y plane from the x-axis toward the y-axis. The elevation angle γ is measured from the x-y plane toward the z-axis. The elevation angle γ, which varies from $-\pi/2$ to $\pi/2$, can be calculated from:

$$\gamma = \sin^{-1} (\mathbf{L} \cdot \mathbf{Z}) \qquad (9\text{–}66)$$

The azimuth angle σ, which varies from 0 to 2π, can be computed from:

$$\sigma = \tan^{-1}\left[\frac{\mathbf{L}\cdot\mathbf{Y}}{\mathbf{L}\cdot\mathbf{X}}\right] \tag{9–67}$$

where the required signs of $\sin\sigma$ and $\cos\sigma$ are those of the dot products in the numerator and denominator, respectively.

9.5.2 AUXILIARY ANGLES AT TRANSMISSION TIME AT A GPS SATELLITE

The **X**, **Y**, and **Z** unit vectors for the transmitting GPS satellite are interpolated from the PV file for the GPS satellite at the transmission time t_2(ET) at the GPS satellite. This interpolation is performed in Step 9 of the spacecraft light-time solution (see Section 8.3.6, Step 9 and Section 7.3.3, Step 3). The unit vector **Z** for a GPS satellite is perpendicular to the reference ellipsoid for the Earth, directed down.

The space-fixed unit vector **L** directed from the transmitting GPS satellite at the transmission time t_2 to the TOPEX satellite or a GPS receiving station on Earth at the reception time t_3 can be calculated from Eq. (9–21), where:

$$\mathbf{L}_{\mathrm{SF}} = \frac{\mathbf{r}_{23}}{r_{23}} \tag{9–68}$$

and

$$\Delta\mathbf{L} = -\frac{\dot{\mathbf{r}}_2^C(t_2)}{c} \tag{9–69}$$

where $\dot{\mathbf{r}}_2^C(t_2)$ is the space-fixed velocity vector of the transmitting GPS satellite relative to the Solar-System barycenter C in that frame of reference and relative to the Earth E in the local geocentric space-time frame of reference.

The auxiliary angles computed at the transmitting GPS satellite are the azimuth σ and elevation γ angles, which are defined the same as for the TOPEX satellite. They can be computed from Eqs. (9–66) and (9–67).